

# A Primer on Digital Beamforming

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March 26, 1998

## Introduction

Beamforming is the combination of radio signals from a set of small non-directional antennas to simulate a large directional antenna. The simulated antenna can be pointed electronically, although the antenna does not physically move. In communications, beamforming is used to point an antenna at the signal source to reduce interference and improve communication quality. In direction finding applications, beamforming can be used to steer an antenna to determine the direction of the signal source.

This introduction to beamforming covers the basic properties of antennas and antenna arrays, then explains how beamformers are built using digital radio hardware and DSP's. Super-resolution direction finding is also explained.

## Antennas and Wavelength

An antenna for a radio transmitter converts electrical signals on a cable, from the transmitter, into electromagnetic waves. The antenna consists of electrical conductors (wires, pipes, reflecting surfaces, etc) that create electric and magnetic fields in the space around them. If the fields are changing, they propagate outward through space as an electromagnetic wave at the speed of light.

$$\text{Speed of Light } c = 3 \times 10^8 \text{ meters/sec}$$

Any antenna that transmits can also receive. Passing electromagnetic waves excite currents in the antenna's conductors. The antenna captures some energy from passing waves and converts it to an electrical signal on the cable.

When designing an antenna, its dimensions are specified in terms of the *wavelength* of the radio signal being transmitted or received. Wavelength is the distance from the beginning of one electromagnetic wave cycle to the next.

$$\lambda = c / f_c$$

$\lambda$  is wavelength in meters

$f_c$  is the carrier frequency of the radio signal in Hz

$c$  is the speed of light ( $3 \times 10^8$  meters/sec)

### Wavelengths For Common Radio Signals

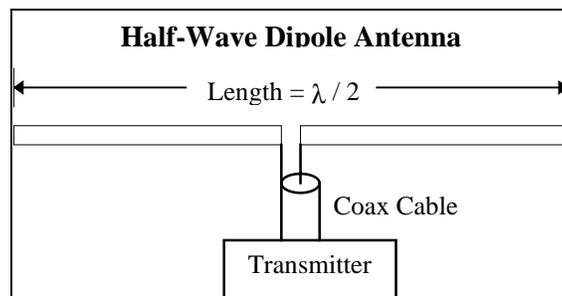
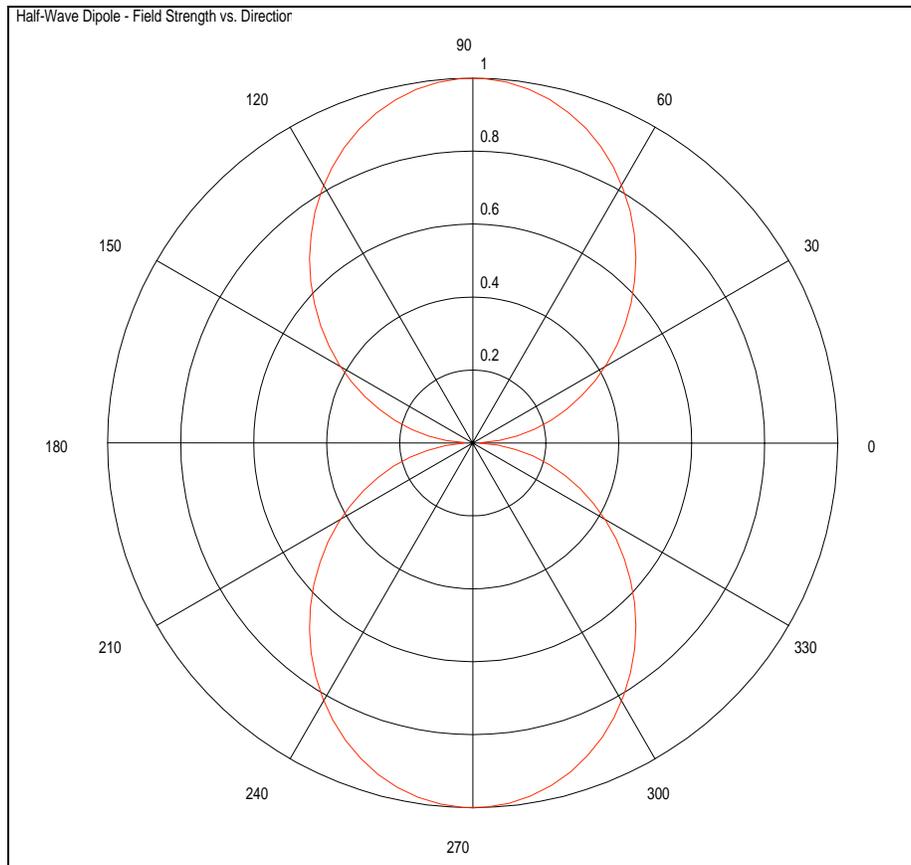
Signal	Frequency	Wavelength
AM Radio	1 MHz	300 meters
FM Radio	100 MHz	3 meters
Cellular Telephone	850 MHz	35 cm
Cellular PCS	1,800 MHz	17 cm
X-Band Radar	10,000 MHz	3 cm

## Antenna Radiation Patterns

A transmitting antenna generates stronger electromagnetic waves in some directions than others. A plot of field strength vs. direction is called the antenna's "*radiation pattern*." It's always the same for receiving as for transmitting.

An electromagnetic wave measured at a point far from the antenna is the sum of the radiation from all parts of the antenna. Each small part of the antenna is radiating waves of a different amplitude and phase, and each of these waves travels a different distance to the point where a receiver is located. In some directions, these waves add constructively to give a gain. In some directions they add destructively to give a loss.

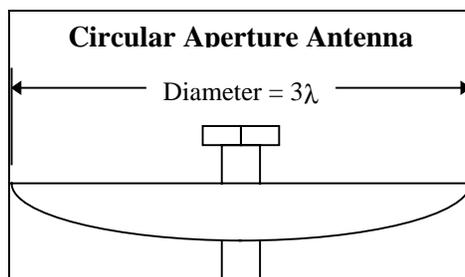
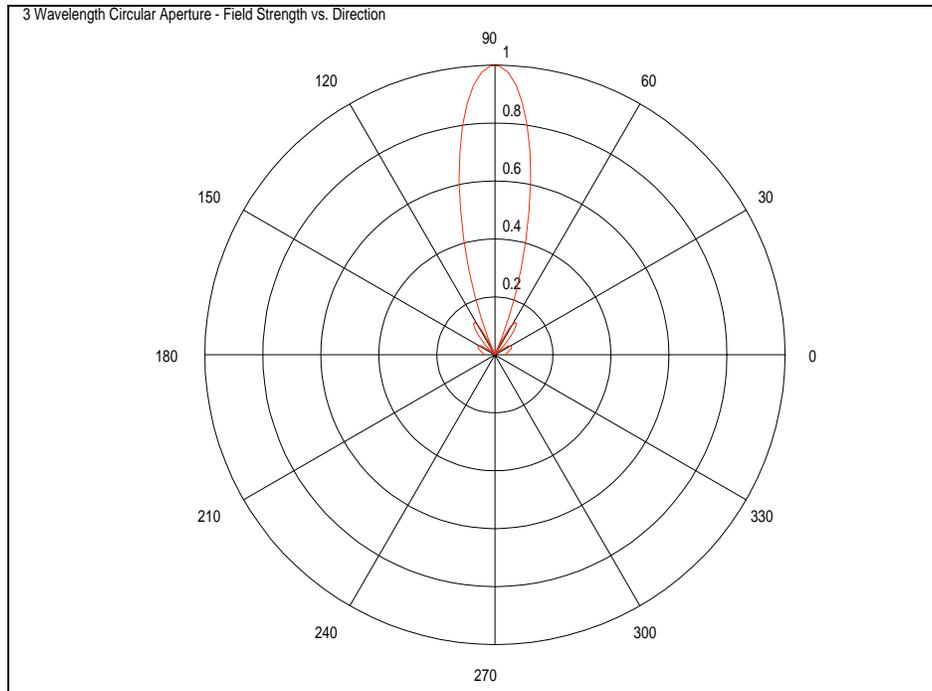
A half-wave dipole is a simple antenna that consists of a half wavelength of wire, cut in the center for connection of the cable. The following figure shows its radiation pattern.



## Directional Antennas

A directional antenna is one designed to have a gain in one direction and a loss in others. An antenna is made directional by increasing its size. This spreads the radiating conductors of the antenna over a larger distance, so that the constructive and destructive interference can be better controlled to give a directional radiation pattern.

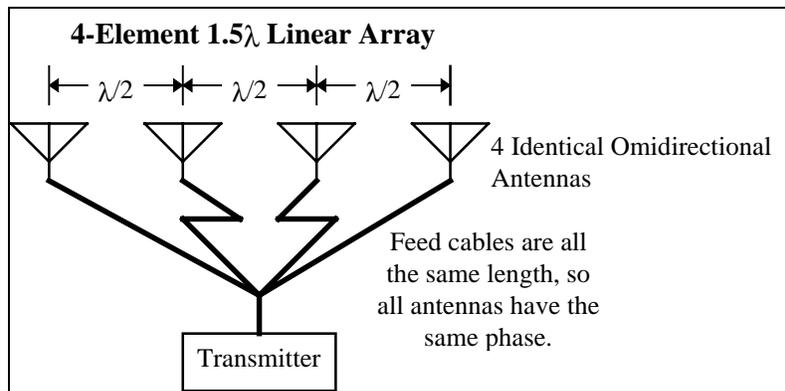
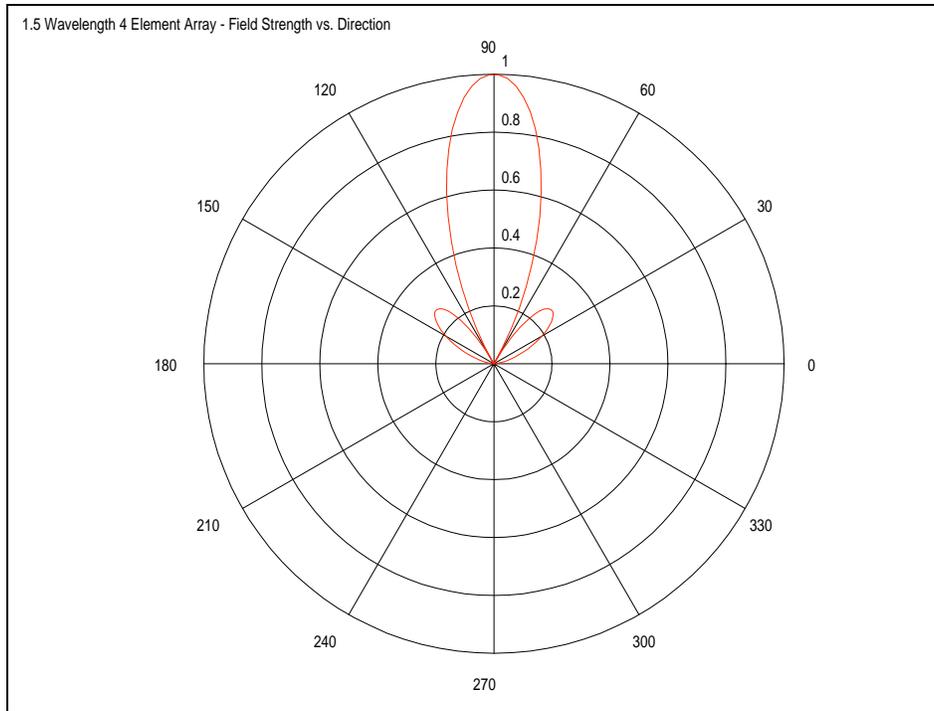
A satellite dish antenna can, simplistically, be considered a circular surface that radiates electromagnetic waves equally from all parts. It has a narrow central “*beam*” of high gain, as shown in the following figure, that is aimed at the satellite. As the dish diameter, in wavelengths, is increased the central beam gets narrower. Notice the smaller beams, called “*side lobes*”, on either side of the central beam. Directions in which the signal strength is zero are called “*nulls*.”



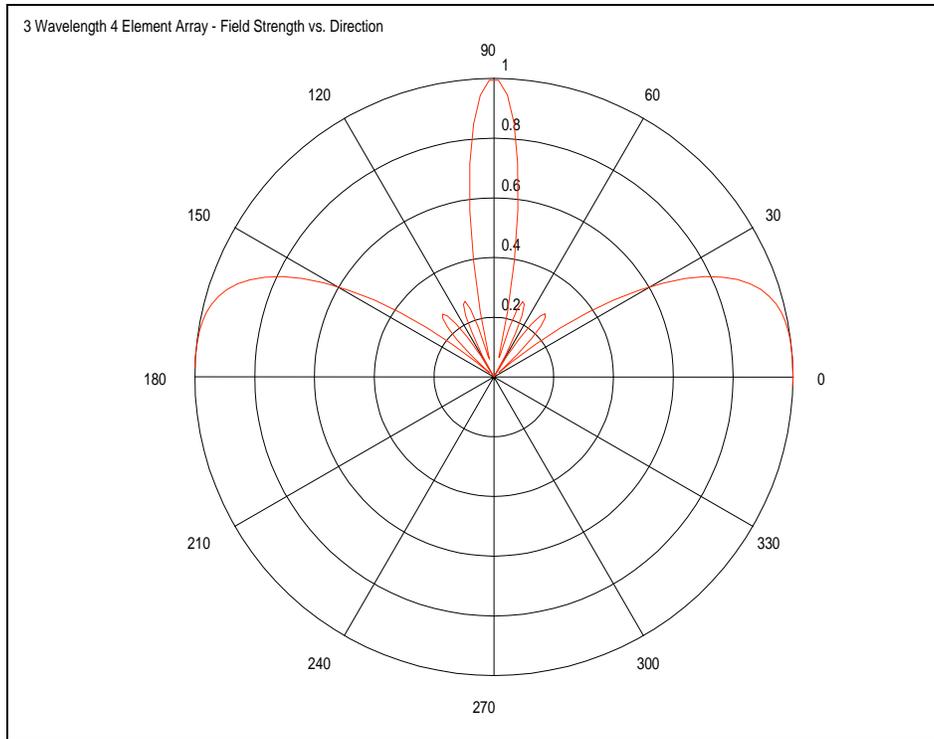
## Linear Arrays

A simple directional antenna consists of a linear array of small radiating antenna elements, each fed with identical signals (the same amplitude and phase) from one transmitter. As the total width of the array increases, the central beam becomes narrower. As the number of elements increases, the side lobes become smaller.

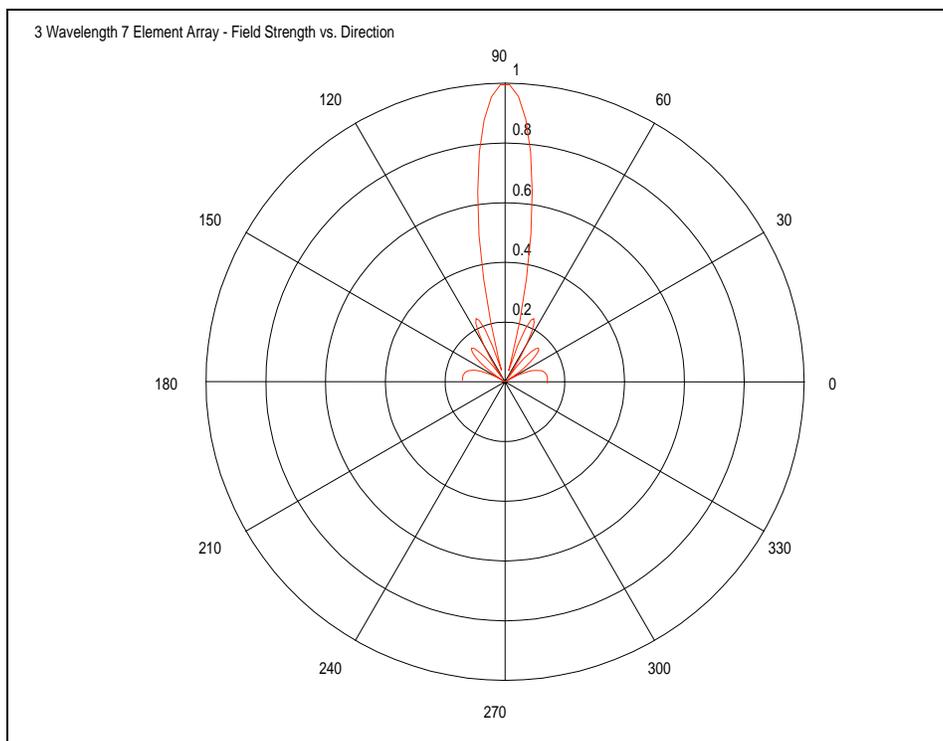
The following figure is the radiation pattern for a line of 4 elements (small antennas) spaced 1/2 wavelength apart.

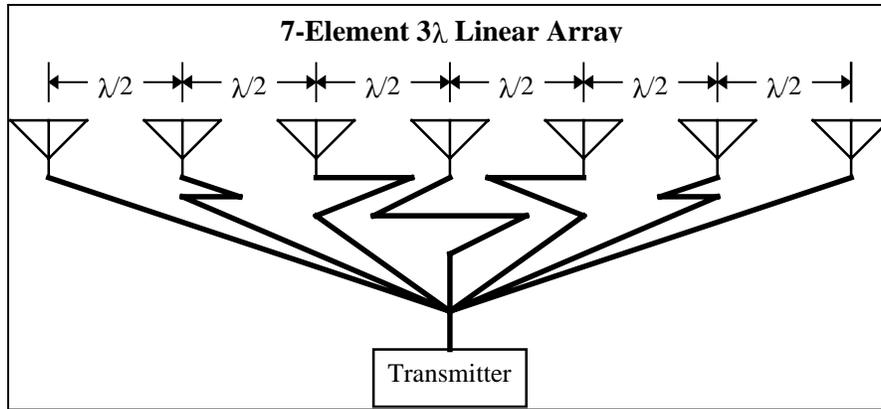


If the spacing is increased to more than  $1/2$  wavelength, large side lobes begin to appear in the radiation pattern. However, the central beam gets narrower because the overall length of the antenna has increased. The following radiation pattern, for 4 elements spaced 1 wavelength apart, illustrates this.



By keeping the overall length the same, and adding elements to reduce the spacing back to  $1/2$  wavelength, the side lobes are reduced. Following is the radiation pattern if 3 more elements are added to the antenna above to reduce the element spacing.

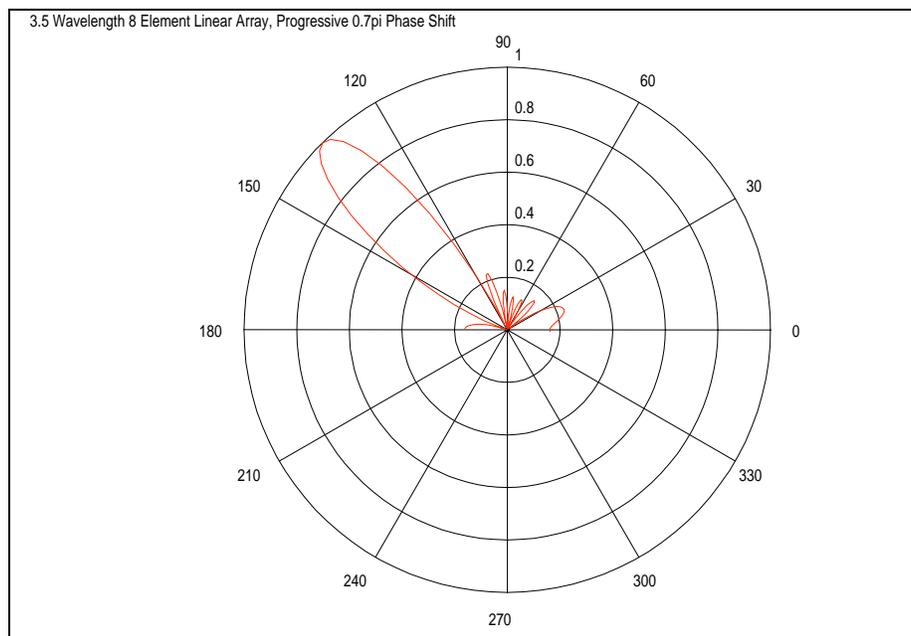


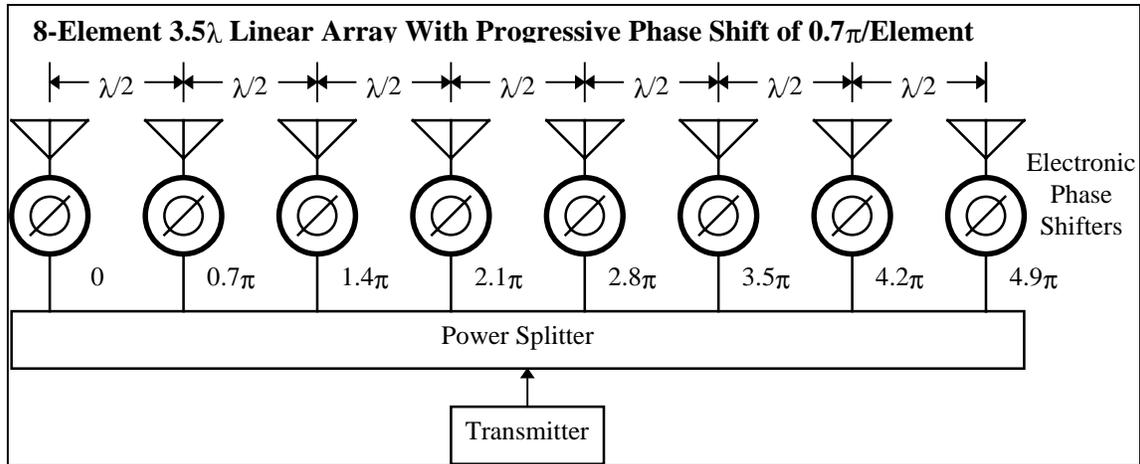


### ***Electronically Steered Arrays***

By varying the signal phases of the elements in a linear array, its main beam can be steered. The simplest way of controlling signal phase is to systematically vary the cable lengths to the elements. Cables delay the signal and so shift the phase. However, this does not allow the antenna to be dynamically steered.

In an electronically steered array, programmable electronic phase shifters are used at each element in the array. The antenna is steered by programming the required phase shift value for each element. The beam pattern below is for an 8-element linear array with a progressive phase shift of  $0.7\pi$  per element. The central beam has been steered about 45 degrees to the left. A phase shift of  $2\pi$  corresponds to one wavelength or one carrier wave period, and more positive values are equivalent to saying that the signal is transmitted earlier.





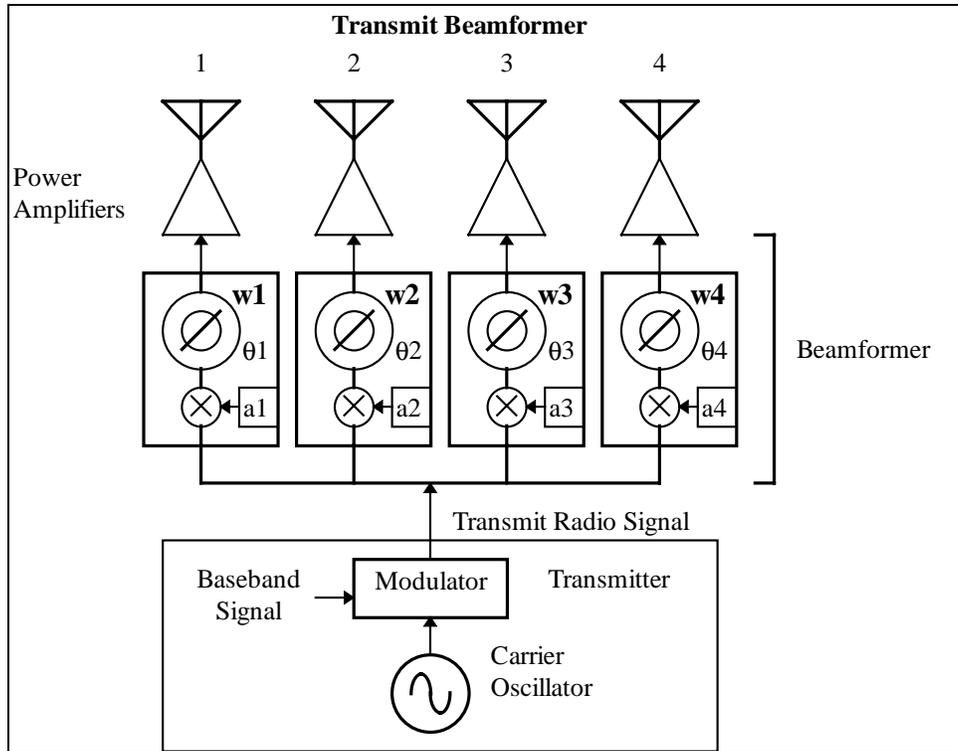
### Array Configurations

An antenna array does not need to be linear. Often, antenna elements are arranged in a circle so that the array can form beams equally well in all directions. On vehicles, antenna elements may be placed in any convenient locations and at different heights to form a 3-dimensional array. For these arrays, determining phase shifts to steer the antenna is more complicated than for linear arrays.

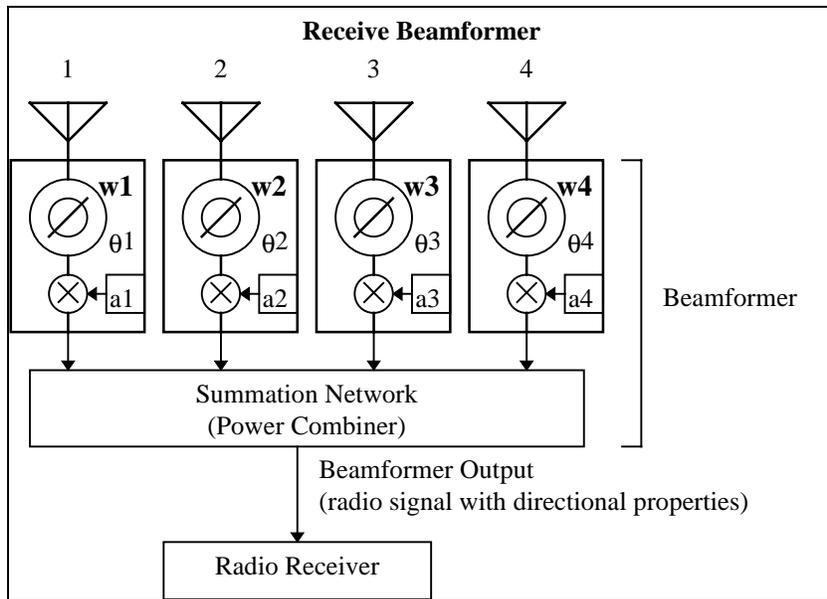
### Beamforming

In beamforming, both the amplitude and phase of each antenna element are controlled. Combined amplitude and phase control can be used to adjust side lobe levels and steer nulls better than can be achieved by phase control alone. The combined relative amplitude  $\mathbf{a}_k$  and phase shift  $\theta_k$  for each antenna is called a “**complex weight**” and is represented by a complex constant  $\mathbf{w}_k$  (for the  $k^{\text{th}}$  antenna).

A beamformer for a radio transmitter applies the complex weight to the transmit signal (shifts the phase and sets the amplitude) for each element of the antenna array.



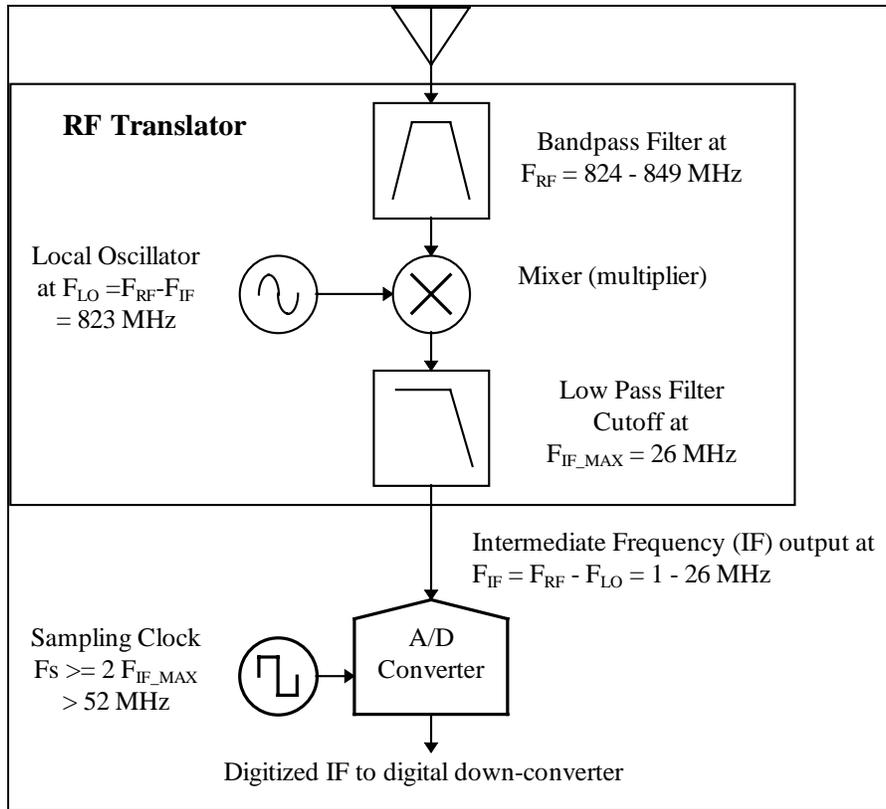
A beamformer for radio reception applies the complex weight to the signal from each antenna element, then sums all of the signals into one that has the desired directional pattern.



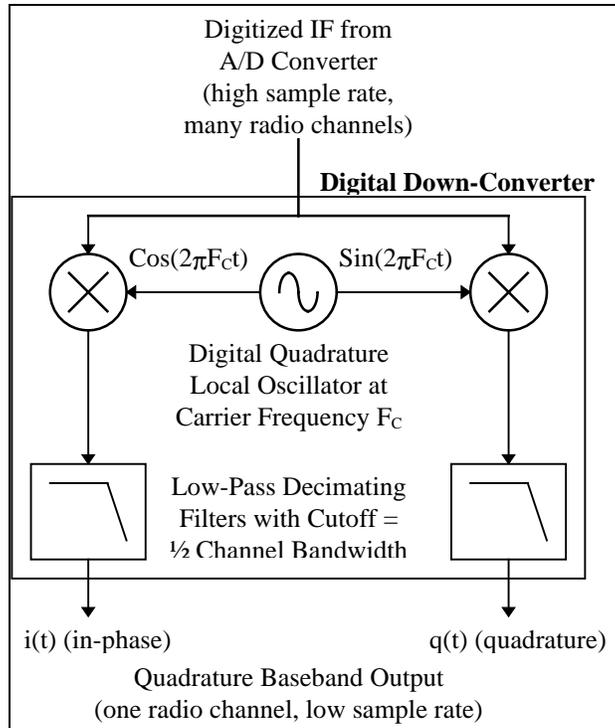
## Digital Beamforming

In digital beamforming, the operations of phase shifting and amplitude scaling for each antenna element, and summation for receiving, are done digitally. Either general-purpose DSP's or dedicated beamforming chips are used.

The rest of this discussion focuses on beamforming receivers. Digital processing requires that the signal from each antenna element is digitized using an A/D converter. Since radio signals above shortwave frequencies ( $>30$  MHz) are too high to be directly digitized at a reasonable cost, digital beamforming receivers use analog “**RF translators**” to shift the signal frequency down before the A/D converters. The following figure shows a translator that shifts the entire cellular telephone uplink band at 824-849 MHz down to the 1-26 MHz range.



Once the antenna signals have been digitized, they are passed to “digital down-converters” that shift the radio channel’s center frequency down to 0 Hz and pass only the bandwidth required for one channel. The down-converters produce a “**quadrature**” baseband output at a low sample rate.



The quadrature baseband  $i$  and  $q$  components can be used to represent a radio signal as a complex vector (phasor) with real and imaginary parts. Two components are required so that both positive and negative frequencies (relative to the channel center frequency) can be represented.

$$\mathbf{s}(t) = \mathbf{x}(t) + j \mathbf{y}(t)$$

$s(t)$  is the complex baseband signal

$\mathbf{x}(t) = \mathbf{i}(t)$  is the real part

$\mathbf{y}(t) = -\mathbf{q}(t)$  is the imaginary part

$j$  is  $\sqrt{-1}$

For beamforming, the complex baseband signals are multiplied by the complex weights to apply the phase shift and amplitude scaling required for each antenna element.

$$\mathbf{w}_k = \mathbf{a}_k e^{j\sin(\theta_k)}$$

$$\mathbf{w}_k = \mathbf{a}_k \cos(\theta_k) + j \mathbf{a}_k \sin(\theta_k)$$

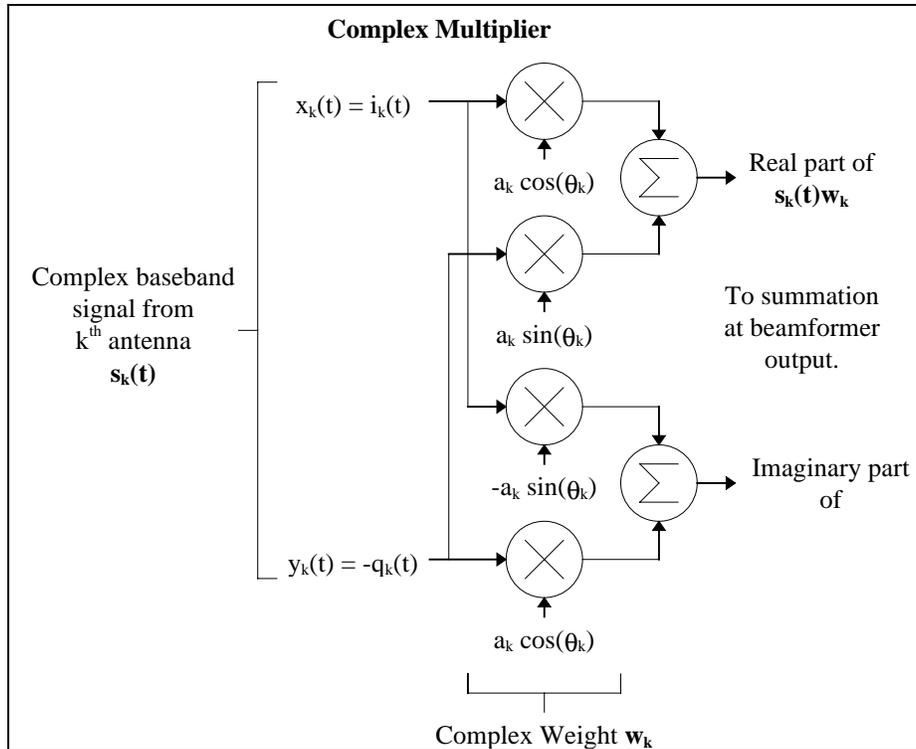
$w_k$  is complex weight for the  $k^{\text{th}}$  antenna element

$a_k$  is the relative amplitude of the weight

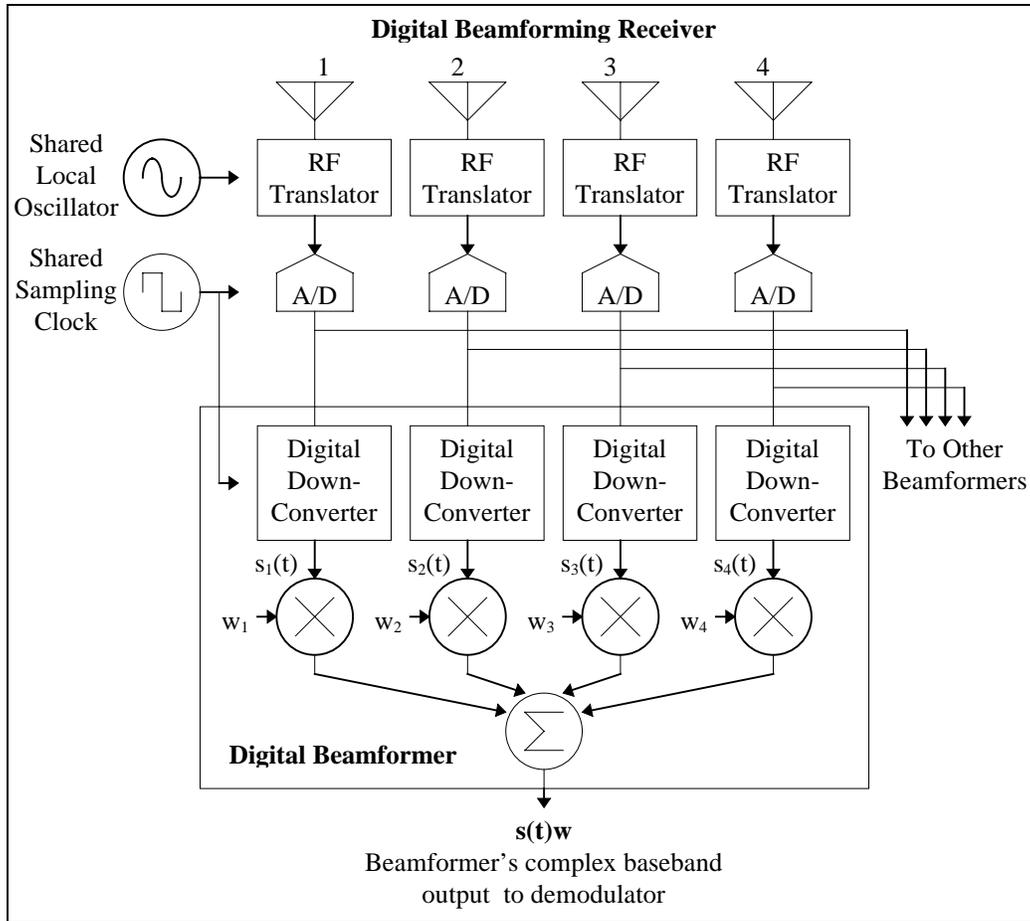
$\theta_k$  is the phase shift of the weight

A general-purpose DSP can implement the complex multiplication for each antenna element:

$$\mathbf{s}_k(t) \mathbf{w}_k = \mathbf{a}_k \{ [\mathbf{x}_k(t) \cos(\theta_k) - \mathbf{y}_k(t) \sin(\theta_k)] + j [\mathbf{x}_k(t) \sin(\theta_k) + \mathbf{y}_k(t) \cos(\theta_k)] \}$$



The following figure shows a complete digital beamforming receiver. One set of antenna elements, RF translators, and A/D converters can be shared by a number of beamformers. All RF translators and A/D converters share common oscillators so that they all produce identical phase shifts of the signal. Within the digital beamformer, all digital down-converters share a common clock, are set for the same center frequency and bandwidth, and their digital local oscillators are in-phase so that all phase shifts are identical. Each DDC's baseband output is multiplied by the complex weight for its antenna element, and the results are summed to produce one baseband signal with directional properties. A demodulator would then follow to recover information from the radio signal.



## Adaptive Beamforming

The complex weights  $w_k$  for the antenna elements are carefully chosen to give the desired peaks and nulls in the radiation pattern of the antenna array. In a simple case, the weights may be chosen to give one central beam in some direction, as in a direction-finding application. The weights could then be slowly changed to steer the beam until maximum signal strength occurs and the direction to the signal source is found.

In beamforming for communications, the weights are chosen to give a radiation pattern that maximizes the quality of the received signal. Usually, a peak in the pattern is pointed to the signal source and nulls are created in the directions of interfering sources and signal reflections.

**Adaptive Beamforming** is the process of altering the complex weights on-the-fly to maximize the quality of the communication channel. Here are some commonly used methods:

**Minimum Mean-Square Error** The shape of the desired received signal waveform is known by the receiver. Complex weights are adjusted to minimize the mean-square error between the beamformer output and the expected signal waveform.

**Maximum Signal-to-Interference Ratio** Where the receiver can estimate the strengths of the desired signal and of an interfering signal, weights are adjusted to maximize the ratio.

**Minimum Variance** When the signal shape and source direction are both known, choose the weights to minimize the noise on the beamformer output.

Often, constraints are placed on the adaptive beamformer so that the complex weights do not vary randomly in poor signal conditions. Some radio signals include “training sequences” so that an adaptive beamformer may quickly optimize its radiation pattern before the useful information is transmitted.

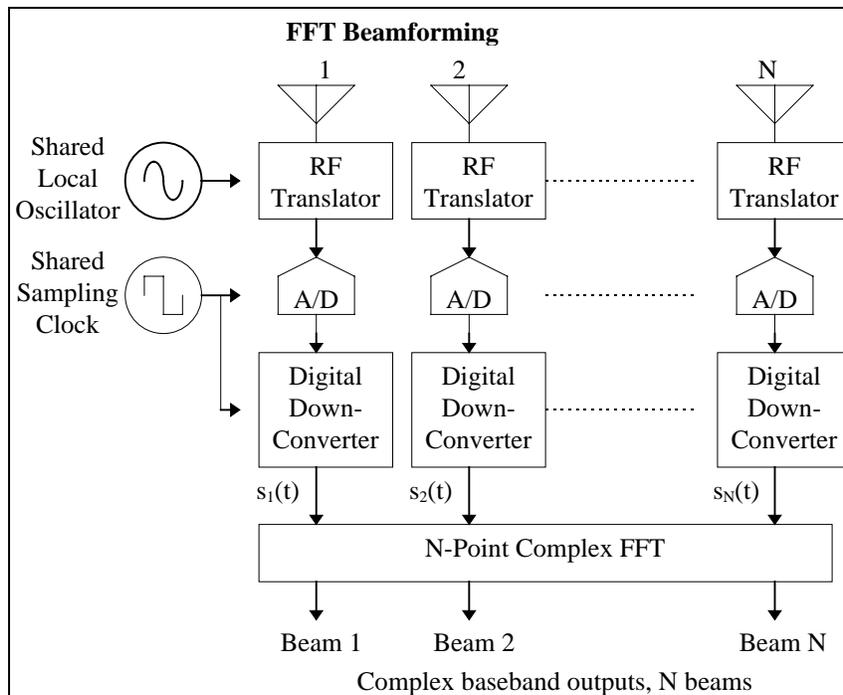
### Smart Antennas

Adaptive beamforming systems for communications are sometimes referred to as “**smart antenna**” systems. For cellular telephone, one base station with a smart antenna system can support more than one user on the same frequency, as long as they are in different directions, by steering individual antenna beams at each user. This is sometimes called “**spatial domain multiple access**” (SDMA). It’s estimated that the capacity of cellular telephone systems can be doubled by using smart antennas.

### FFT’s in Beamforming

In digital beamforming, many beamformers can share one set of antenna elements, rf translators, and A/D converters. The beamformers may have their central beams pointed in different directions. In situations where a fixed set of non-overlapping beams must be formed simultaneously (radar, sonar, direction-finding) an FFT can implement many beamformers efficiently.

The following figure shows an FFT beamformer with N antenna elements. Each element requires a digital down-converter. All DDC’s produce a baseband sample simultaneously, and all of these are passed at once to an N-point complex FFT. The FFT then produces a set of N complex outputs, each of which is the next baseband sample for a different beam.



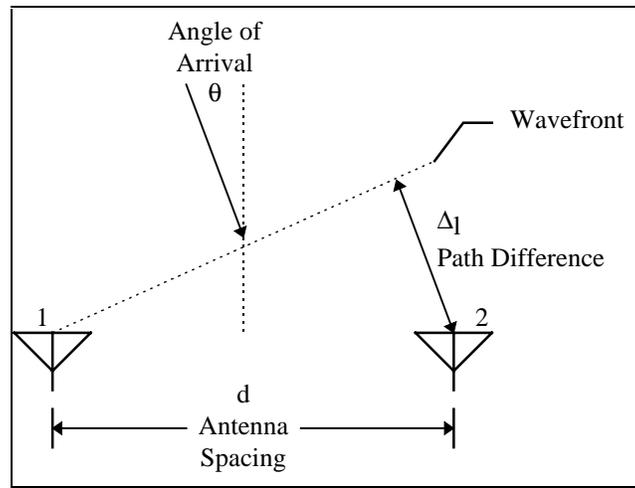
In this case, a “**spatial FFT**” is being performed: The FFT is processing a set of samples that are separated in space (not in time). Therefore, its outputs are a set of samples that are separated in direction (not in frequency).

FFT beamforming as shown above is not flexible. For a linear array, the N beams are fixed and equally spaced in direction. They range from -90 to +90 degrees from broadside of the array. The beams are orthogonal: the central peak of any beam lies in a null on all other beams. Such a set of beams is useful for radar mapping, but not very useful for communications.

It is possible to use FFT's for beamforming in communications. A set of FFT outputs can be combined, using complex weights and sums as before, to form arbitrary radiation patterns. This is called "**beam-space beamforming.**" The previous approach of combining baseband signals from different antenna elements is called "**element-space beamforming.**"

### **Super-Resolution Direction Finding**

The term "**super-resolution**" applies to the ability to measure the angle of arrival of a radio signal with much higher resolution than the beam width of the antenna array. The method requires accurately measuring the phases of the signals from the array elements and, from these, calculating the angle of arrival.



A wavefront from direction  $\theta$  arrives at antenna 1 first. Then, after travelling an additional path distance  $\Delta l$  it arrives at antenna 2.

$$\Delta l = d \sin \theta$$

The path difference results in a phase difference  $\Delta \phi$  between the signals from the two antennas:

$$\Delta \phi = 2\pi \Delta l / \lambda$$

$$\Delta \phi = 2\pi d \sin \theta / \lambda$$

A direction-finding system calculates the angle of arrival from the phase difference:

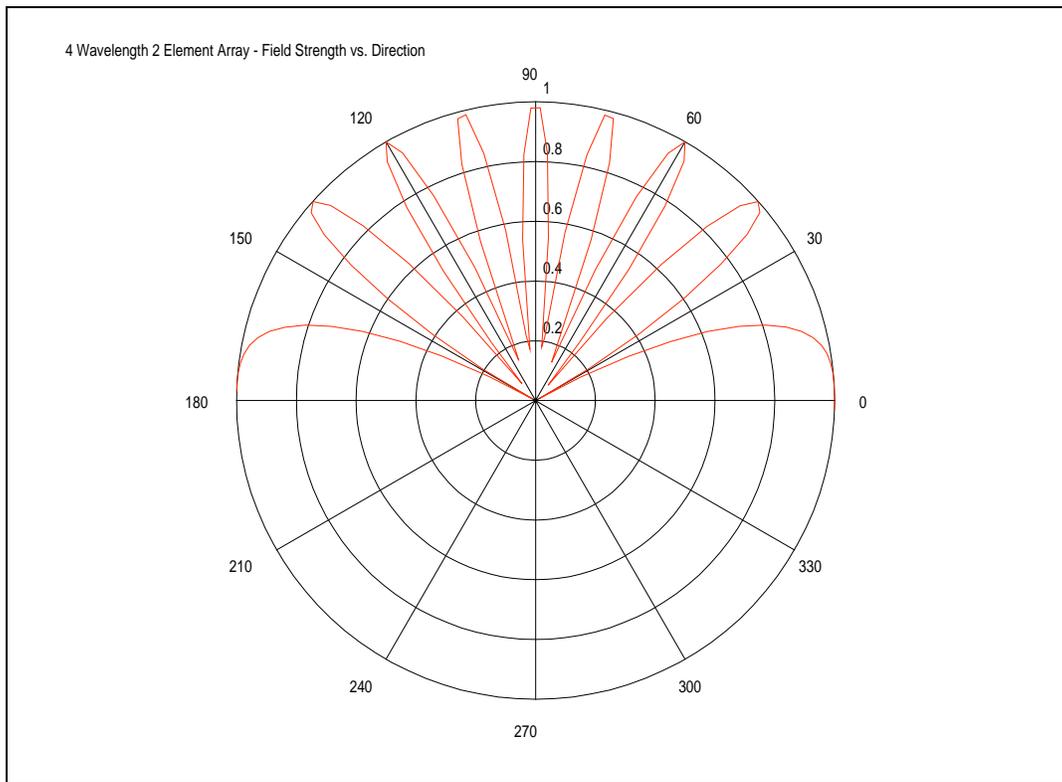
$$\theta = \sin^{-1} (\Delta \phi \lambda / (2\pi d))$$

For a super-resolution result to be accurate, the arriving wave must be a direct signal from the source - a "**plane wave**" with a straight wavefront. Signal reflections (multipath) and interfering signals cause super-resolution systems to fail. A super-resolution system cannot operate if two or more signal sources share the same frequency, since the receiver's output phase no longer reflects the phase of an incoming plane wave.

Beamforming can be used for direction finding by rotating the central beam of an array to give maximum received signal strength. With this method, the angular resolution is limited by the beam width produced by the beamformer. Also, false measurements will occur if a side lobe is mistakenly steered to the signal source, instead of the array's central lobe. However, it is possible to measure the directions of multiple sources and to identify the directions of reflections with a beamforming system.

For two antenna elements spaced at 4 wavelengths, the following diagram shows the radiation pattern that a beamformer would produce. The main drawback of the beamforming approach - many side lobes unless

many antenna elements are used - is apparent. A super-resolution system using the same two antennas could measure direction accurately, provided that the only an undistorted plane wave is arriving.



## References

John Litva and Titus Kwok-Yeung Lo, *Digital Beamforming in Wireless Communications*, Artech House, Norwood, MA, 1996.

Warren L. Stutzman and Gary A. Thiele, *Antenna Theory and Design*, John Wiley & Sons, New York, 1981.